Answers to Coursebook questions – Chapter E3

1 See discussion on page 506 in *Physics for the IB Diploma*.
The method fails for stars far away (more than about 300 pc or 1000 ly) because then the parallax angle is too small to be measured accurately.

2 **a** Apparent magnitude is a measure of the brightness of the star, on a logarithmic scale, as it appears from earth.

b Absolute magnitude is the apparent magnitude a star would have when observed from 10 pc away.

3 The distance is $\frac{1}{0.285} = 3.51\text{ pc}$.

4 The distance is $\frac{10.8}{3.26} = 3.31\text{ pc}$ and so the parallax angle is $\frac{1}{3.31} = 0.302''$.

5 **a** The distance is $\frac{1}{0.0067} = 149\text{ pc}$.

b The diameter is
$$D = d\theta = 149 \times \frac{0.016}{3600} \times \frac{\pi}{180}\text{ pc} = 149 \times \frac{0.016}{3600} \times \frac{\pi}{180} \times 3.26 \times 9.46 \times 10^{15} = 3.56 \times 10^{11}\text{ m.}$$
And so the radius is $\frac{3.56 \times 10^{11}}{2} = 1.78 \times 10^{11}\text{ m}$.
This is about 256 times larger than the radius of the sun.

6 The distance is $\frac{1}{0.025} = 40\text{ pc}$.
The actual distance is greater than 10 pc and so the star *appears* dimmer than the equivalent of magnitude 0.8.
Hence its apparent magnitude is greater than 0.8.
Or, from $m - M = 5\log\frac{d}{10}$, we get $m = 0.8 + 5\log 4 = 3.8$.

7 **a** The distance is $\frac{1}{0.250} = 4.00\text{ pc}$.

b It is dimmer than the limit of $m = 6$ and so cannot be seen by the naked eye.

8 The stars differ by $\Delta M = 2$ and so have a luminosity ratio of $100^{2/5} = 6.3$.

9 The stars differ by $\Delta M = 1.1$ and so have a luminosity ratio of $100^{1.1/5} = 2.8$ with Cappella being the brighter of the two.

10 **a** Star A appears brighter because its apparent magnitude is smaller.

b The distance of star A is larger since its parallax is smaller. Since it appears brighter and it is further away it must have a larger luminosity than star B.

11 **a** The luminosity is the same since the absolute magnitude is the same.

b Star B has a larger parallax, so it is closer. Hence it appears brighter.

12 Since the stars are part of a binary they have roughly the same distance from us. Hence what appears brighter (star A in this case because of the smaller apparent magnitude) is intrinsically brighter.

13 The temperature is found from $\lambda T = 2.90\times10^{-3} \Rightarrow T = \dfrac{2.90\times10^{-3}}{2.42\times10^{-7}} \approx 12{,}000\ \text{K}$.

From the HR diagram a main sequence star at this temperature has a luminosity that is **about 100 times larger** than the Sun's, i.e. $3.9\times10^{28}\ \text{W}$.

Then, $b = \dfrac{L}{4\pi d^2} \Rightarrow d = \sqrt{\dfrac{L}{4\pi b}} = \sqrt{\dfrac{3.9\times10^{28}}{4\pi\times8.56\times10^{-12}}} = 1.9\times10^{19}\ \text{m} = 2.0\times10^{3}\ \text{ly}$.

14 The distance to the star is $\dfrac{1}{0.034} = 29.4\ \text{pc} = 29.4\times3.09\times10^{16} = 9.08\times10^{17}\ \text{m}$.

The apparent brightness is then $b = \dfrac{L}{4\pi d^2} = \dfrac{2.45\times10^{28}}{4\pi\times(9.08\times10^{17})^2} = 2.4\times10^{-9}\ \text{Wm}^{-2}$.

The easiest way to find the apparent magnitude is to use a formula that is not on the syllabus: $m = -\dfrac{5}{2}\log\dfrac{b}{b_0} = -\dfrac{5}{2}\log\dfrac{2.36\times10^{-9}}{2.52\times10^{-8}} = 2.57$.

But the answer can be found within what is in the syllabus in a somewhat harder way: the absolute magnitude is found from $m - M = 5\log\dfrac{d}{10}$,

i.e. $M = m - 5\log\dfrac{d}{10}\ M = 2.57 - 5\log\dfrac{29.4}{10} = 0.228$.

In bringing the star to a distance of 10 pc its brightness would increase by a factor of $(\dfrac{29.4}{10})^2 = 8.644$.

Hence, $8.644 = 2.512^{\Delta m} \Rightarrow \Delta m = 2.342$.

Hence the apparent magnitude is $m = 2.342 + 0.228 = 2.57$.

15 **a** We use the non-syllabus formula:

$$\frac{b}{b_0}=100^{-m/5}=100^{-1/5}=0.398 \Rightarrow b=0.398\times 2.52\times 10^{-8}=1.003\times 10^{-8}\approx 1.0\times 10^{-8}\text{ W m}^{-2}.$$

b The ratio in apparent brightness between Procyon and Altair is $\frac{1.78}{1.003}=1.77$ (Procyon being the brighter of the two).

Hence, $1.77=2.512^{\Delta m} \Rightarrow \Delta m=0.6199\approx 0.62$.

Hence the apparent magnitude of Procyon is $1-0.62=0.38$.

Equivalently, we may use the non- syllabus formula:

$$m=-\frac{5}{2}\log\frac{b}{b_0}=-\frac{5}{2}\log\frac{1.78\times 10^{-8}}{2.52\times 10^{-8}}=0.38.$$

16 Assuming a luminosity of 3500 solar luminosities (the graph is hard to read), we find $b=\frac{L}{4\pi d^2} \Rightarrow d=\sqrt{\frac{L}{4\pi b}}=\sqrt{\frac{3500\times 3.9\times 10^{26}}{4\pi\times 3.45\times 10^{-14}}}=1.77\times 10^{21}\text{ m}=1.9\times 10^{5}\text{ ly}$.

17 **a** From $m-M=5\log\frac{d}{10}$ we have that $0.1-(-7.0)=5\log\frac{d}{10}$ and so $0.1-(-7.0)=5\log\frac{d}{10} \Rightarrow d=10\times 10^{\frac{7.1}{5}}=263\approx 260\text{ pc}$.

b The distance is $\frac{1}{0.760}=1.32\text{ pc}$.

Hence $(-0.27)-M=5\log\frac{1.32}{10}$ and so $M=4.13\approx 4.1$.

c We present two approaches.

First we can find the absolute magnitude using the distance to the sun $1.0\text{ AU}=4.86\times 10^{-6}\text{ pc}$ as $(-26.74)-M=5\log\frac{4.86\times 10^{-6}}{10}$ i.e. $M=4.83$.

Then the distance at which the apparent magnitude becomes the limiting $m=6.0$ is $6.0-4.83=5\log\frac{d}{10} \Rightarrow d=10\times 10^{\frac{1.17}{5}}\approx 17\text{ pc}$.

Alternatively, we can see the sun when the apparent magnitude does not exceed the limit of $m=6.0$.

Then $\Delta m=32.74$ and so at the larger distance the apparent brightness will drop by $100^{32.74/5}=1.25\times 10^{13}$.

The distance will then be larger than the actual sun distance by a factor of $\sqrt{1.25\times 10^{13}}=3.5\times 10^{6}$, i.e. at a distance of 3.5×10^{6} AU or about 17 pc.